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NONLINEAR ESTIMATION OF TIME-VARYING PARAMETERS OF POWER SYSTEM SIGNALS

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RESUMEN.

En este artículo se proponen y discuten tres métodos para calcular la transformada de Hilbert con el fin de identificar y caracterizar el comportamiento dinámico no lineal y no estacionario de la señal del sistema de potencia en tiempo real mediante su frecuencia instantánea.

Los enfoques se basan en la transformada de Hilbert-Huang (HHT), que emplea la descomposición empírica modal (EMD) y la transformada de Hilbert (HT) en la señal analítica para estimar la frecuencia instantánea.

El objetivo de este estudio es calcular la transformada de Hilbert de una señal real buscando minimizar los errores inicial y final de la serie de datos de la señal del sistema de potencia para obtener una óptima estimación de la frecuencia instantánea.

Se emplean datos recolectados de un sistema interconectado. Se proporciona la comparación de los resultados numéricos obtenidos por los métodos propuestos con el fin de determinar el enfoque óptimo.

Palabras Clave: señal de potencia, transformada de Hilbert-Huang, convolución, frecuencia instantánea.

ABSTRACT.

In this paper three different methods for computing the Hilbert transform are proposed and discussed with the purpose of the identification and characterization of the nonlinear and non-stationary dynamic behavior of real-time power system signal through its instantaneous frequency.

The approaches are based on the Hilbert-Huang transform (HHT), which employs the empirical mode decomposition (EMD) and Hilbert transform (HT) in the analytic signal to estimate the instantaneous frequency.

The objective of this study is the computing the Hilbert transform of a real signal and with the development of these methods to minimize the error in the initial and the end of the data-time of power system signal to get a optimal estimation of the instantaneous frequency.

Field data, collected at an interconnected power system are used in this comparative analysis. The comparison of numerical results obtained by the proposed methods is provided for determinate the optimal approach.

Keywords: power signal, Hilbert-Huang transform, convolution, instantaneous frequency.

1. INTRODUCTION

Identification and characterization of nonlinear and non-stationary signals in the field of the power system dynamics has been a great development in recent times [1-3]. Disturbances presented during the operation of an electric power system appear as nonlinear and non-stationary phenomena and knowing their behavior is possible to implement control actions.

The Hilbert-Huang transform (HHT), method introduced by Huang et al (1998) [9], is a technique effective in decomposing time-series data nonlinear and non-stationary and is a great tool for this study. When transient phenomena occur, is possible to realize a precise analysis on the dynamic behavior of measured data. This method analyzes complex temporal dynamics on a real-time basis.

The HHT of a real signal is decomposing in several intrinsic mode functions (IMFs), which has particular physical meaning and each of these IMFs is a simple oscillatory signal. When the original signal is decomposed into a set of IMFs, the Hilbert transform is applied to each component to define the instantaneous characteristics: amplitude, phase and frequency.

When computing data of the instantaneous characteristics of each IMF the estimation of instantaneous frequency depends largely on the method used to compute the Hilbert transform because it produces mistakes at the extremes of data recording and the estimated values are unreliable. Mistakes are manifested as fluctuations and they comprise a poor performance in the instantaneous feature analyzed [4].

The three methods are focused in the implementation of the Hilbert transform in power signal for the estimation of instantaneous frequency. These methods are: the Time-domain approach, the Frequency-domain approach (Fourier method) and the Convolution method. They propose changes in the implementation of the Hilbert transform to get a best estimation itself.

The aim of this work is to obtain the best approximation of the Hilbert transform and thus gain a better estimation of the instantaneous frequency eliminating the mistakes in the extremes of data recording. A case of study is presented to illustrate the results of the application of the different methods proposed.

2. THEORICAL FRAMEWORK

The power system signals are multi-components, nonlinear and non-stationary and require to be decomposed in mono-component signals with a simple mode of oscillation for her analysis. Therefore is required to use the Hilbert-Huang transform.

The Hilbert-Huang transform is a time-frequency representation method composed by two steps, the first step is the empirical mode decomposition, and the second step is the Hilbert transform. The methods proposed a modification performed in the second step for to compute the Hilbert transform of an input signal, the results are compared and an efficient estimation is obtained.

HHT is a powerful method for time-frequency analysis of nonlinear and non-stationary data [7]. It is based on an adaptive basis, and the frequency is defined through the Hilbert transform. The comparisons with traditional methods as Fourier shows that the HHT allows the analysis of nonlinear and non-stationary signals.

2.1. Empirical mode decomposition (EMD).

The empirical mode decomposition (EMD) technique is a systematic method for numerically decomposing any time equally spaced time series, $x(t)$ into its own intrinsic mode functions, i.e. the IMFs.

The purpose of this method is to decompose multi-component, non-stationary and nonlinear signals into a finite number of simple oscillatory signals. Each one of these mono-component and oscillatory signals are represented by an Intrinsic Mode Function (IMF). An IMF is defined as a time series which satisfies three critical requirements:

a) In the whole data set, the number of extrema and the number of zero-crossing must either equal or differ at most by one.

b) At any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

c) The linear superposition of all IMFs should reconstruct the time series.

The IMFs are obtained through a progressive sifting process with cubic spline interpolation and its algorithm for decompose a signal $x(t)$ into IMFs is as follows:

1. Starting with the original signal $x(t)$, set $r_0(t) = x(t)$, and $j = 1$.

2. Extract the jth IMF using the sifting procedure

a. Set $h_0(t) = r_j(t)$ and $i = 1$.

b. Identify all extrema of $x(t)$ for $h_0(t)$.

c. Interpolate between maxima (minima) to obtain an envelope $e_{\max}(t)$ $\left(e_{\min}(t)\right)$.

d. Compute the running mean of the envelopes $m_{i-1}(t) = \left[e_{\max_{i-1}}(t) + e_{\min_{i-1}}(t)\right]/2$, and substract it from $h_{i-1}(t)$; determine a new estimate $h_i(t) = h_{i-1}(t) - m_{i-1}(t)$, such that $e_{\min_{i-1}}(t) \le h_i(t) \le e_{\max_{i-1}}(t)$ for all t. Set $i = i + 1$

The step 2(d) is then repeated until $h_i(t)$ satisfies a predetermined stopping criterion, and the first IMF component from the data is called $c_i(t) = h_i(t)$.

3. Obtain an improved residue $r_j(t) = r_{j-1}(t) - c_j(t)$. Repeat steps 2(a) through 2(d) with $j = j + 1$ until the number of extremes in $r_j(t)$ is less than two.

The sifting process has two purposes: to eliminate riding waves and to make the wave-profiles more symmetric.

This method is intuitive, direct, *a posteriori* and adaptive, with the basis of the decomposition based on, and derived from, the data.

Each IMF involves only one mode of oscillation, where the first component (IMF_1) contain the highest frequency oscillation; the frequency content then decrease with the increase in IMF and the last component is the residue ($r_n(t)$).

The stopping criterion of the sifting process is given by the residue $r_n(t)$, becomes a monotonic function from which no more IMF can be extracted, or can still be different from zero mean.

Finally, the signal $x(t)$ is written as the sum:

$$x_j(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) \tag{1}$$

where *n* is the total number of IMFs and $r_n(t)$ is the non-oscillatory residual at the end of the sifting process.

We can rewrite the signal $x(t)$ in the form

$$x_j(t) = \sum_{j=1}^{n} c_j(t) + r_n(t) = \sum_{j=1}^{p} c_j(t) + \sum_{k=p+1}^{n} c_k(t) + r_n(t) \tag{2}$$

where the terms $c_j(t)$ contain the physical behavior of interest, and the remaining *n-p* terms contain uninteresting non-sinusoidal characteristics.

When $x(t)$ is decomposed into a set of IMFs, the Hilbert transform is applied to each component to define the instantaneous characteristics: amplitude, phase and frequency.

2.2. The Hilbert Transform (HT).

Once the EMD technique has been applied to decompose the input signal into a set of IMFs and a residual signal, one may

now analyze the components to extract instantaneous amplitude, phase and frequency information.
The Hilbert transform (HT) of a signal $x(t)$ is defined as [5]

$$\tilde{c}_j(t) = \frac{1}{\pi} P\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau} d\tau \tag{3}$$

in which P indicates the Cauchy principal value.
Application of the HT to the *j*th IMF yields the complex trace representation or analytic signal, $z_j(t)$

$$\begin{gathered} z_j(t) = c_j(t) + i\tilde{c}_j(t) = A(t)e^{i\varphi(t)} \\ c_j(t) = A(t)\cos(\varphi(t)), \quad \tilde{c}_j(t) = A(t)\sin(\varphi(t)) \end{gathered} \tag{4}$$

where $A_j(t)$, $\varphi_j(t)$, and $\omega_j(t)$ are the instantaneous value of amplitude or envelope, phase and frequency of *jth* IMF, respectively.

Through equation (3) in its polar form, it is possible to define the instantaneous characteristics

$$\begin{gathered} A_j(t) = \sqrt{c_j(t)^2 + \tilde{c}_j(t)^2}, \\ \varphi_j(t) = \tan^{-1}\frac{\tilde{c}_j(t)}{c_j(t)}, \\ \omega_j(t) = \frac{d\varphi_j(t)}{dt}, \end{gathered} \tag{5}$$

which are calculated as functions of time and can therefore be related to all temporal events during the measurement.

The Hilbert transform is usually computed in the frequency-domain through the Fourier transform according to the convolution theorem [8]. From equation (3), the Hilbert transform can be rewritten in the form of a convolution, or a linear filter as

$$\tilde{c}_j(t) = c_j(t) * \frac{1}{\pi t} \tag{6}$$

where $1/\pi t$ is the kernel of the transformation.

2.3. Local energy of IMFs.

The instantaneous amplitude $A_j(t)$ and instantaneous frequency $\omega_j(t)$ derived from the IMFs through the Hilbert transform can give us a full energy-frequency-time distribution of the data. The local energy of *jth* IMF is given by

$$E_j(t) = \frac{1}{2} A_j^2(t) \tag{7}$$

Therefore, the IMF with most energy will be the IMF dominant and the IMF with less energy is the residue.

3. DESCRIPTION OF PROPOSED APPROACHES.

Existing approaches to the numerical calculation of the Hilbert transform are based on the computation of the analytic signal using the Fourier transform and are not well adapted for estimation of instantaneous characteristics of a real signal producing numerical errors in the computing of instantaneous parameters in the initial and end data points.
This study presents and contrasts three methods for computing the Hilbert transform of arbitrary signals. These are: a time-domain approach, a frequency-domain approach based on Fourier analysis and a convolution-based method.

3.1. Method I: Frequency-domain approach.

The Hilbert transform is usually computed in the frequency-domain through the Fourier transform according to the convolution theorem. The Hilbert transform can be considered as the convolution given by (6).
This representation of the Hilbert transform as a convolution leads to an alternative way to compute the Hilbert transform in the frequency domain via Fourier transform.
The definition of Cauchy principal value in the time domain is difficult to calculate, so the Hilbert transform in the frequency-domain is defined. Assume $X(f)$ and $\hat{X}(f)$ are the Fourier transform of $x(t)$ and $\hat{x}(t)$, then $X(f)$ and $\hat{X}(f)$ are defined by

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt \ ; \ \omega = 2\pi f \tag{8}$$

$$\hat{X}(f) = -j\,\mathrm{sgn}(f)X(f) \tag{9}$$

where f is called Fourier frequency with units 1/units of the independent variable. The signum function is defined by

$$\mathrm{sgn}(f) = \begin{cases} 1, & for \quad f > 0, \\ 0, & for \quad f = 0, \\ -1, & for \quad f < 0. \end{cases} \tag{10}$$

and the operator $-j\,\mathrm{sgn}(f)$ does not change the amplitude spectrum, but only all spectral components of the original signal are shifted by $\pi/2$ or 90-degrees.

Applying the Fourier transform to the convolution in equation (6), we obtained the Hilbert transform in the frequency domain given by

$$\hat{Y}(j\overline{\omega}) = FT[\hat{y}(t)] = FT[1/\pi t]Y(j\overline{\omega}) = -j\,\mathrm{sgn}(\overline{\omega})Y(j\overline{\omega}) \tag{11}$$

where FT is the Fourier transform.

Assume x_k a real signal, its HT $\hat{x}_k$ can be computed using the fast Fourier transform (FFT) techniques as

$$\hat{x}_k = FFT^{-1}[-j\,\mathrm{sgn}(\overline{\omega})FFT[x_k]] \tag{12}$$

where FFT is the fast Fourier transform, FFT^{-1} is the inverse fast Fourier transform, $\overline{\omega}$ is the *n*th frequency of the discrete Fourier transform, and sgn is the sign function.

Therefore the Hilbert transform is a $-\pi/2$ phase shifter when observed as a linear system whose input is x_k and output is $\hat{x}_k$.

3.2. Method II: Time-domain approach.

Consider a real signal $x(t)$. Assume further that the signal $x(t)$ has been sampled every Δt second to give the sequence $x_k = x(k\Delta t),\ k = 1,2,3,\ldots,n$ and that the Hilbert transform signal $\hat{x}(k)$ can be computed. If the signal $x(t)$ is assumed to vary linearly during the sampling interval [Δt to $n\Delta t$], its Hilbert transform at time Δt can be written as [10]

$$\hat{x}_k = \hat{x}(k\Delta t) = \frac{1}{\pi}\int_{\Delta t}^{N\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau$$

$$= \frac{1}{\pi}\left(\int_{\Delta t}^{2\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau + \ldots + \int_{(k-2)\Delta t}^{(k-1)\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau + \int_{(k-1)\Delta t}^{k\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau\right.$$

$$\left. + \int_{k\Delta t}^{(k+1)\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau + \int_{(k+1)\Delta t}^{(k+2)\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau + \ldots + \int_{(N-1)\Delta t}^{N\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau\right)$$

$$= \frac{1}{\pi}\left(\sum_{i=1}^{k-2} I_i^{(f)} + I_k^{(c)} + \sum_{i=k+2}^{N} I_i^{(r)}\right), \qquad (13)$$

where

$$I_i^{(f)} \equiv \int_{i\Delta t}^{(i+1)\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau \qquad (14)$$

$$I_k^{(c)} \equiv \int_{(k-1)\Delta t}^{k\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau + \int_{k\Delta t}^{(k+1)\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau \qquad (15)$$

$$I_i^{(r)} \equiv \int_{(i-1)\Delta t}^{i\Delta t} \frac{x(\tau)}{k\Delta t - \tau} d\tau \qquad (16)$$

The results given in [9] relative to numerical computation are

$$I_i^{(f)} = x_i \ln\frac{k-i}{k-i-1} + (x_{i+1} - x_i)\left(-1 + (k-i)\ln\frac{k-i}{k-i-1}\right) \qquad (17)$$

$$I_k^{(c)} = x_{k-1} - x_{k+1} \qquad (18)$$

$$I_i^{(r)} = x_i \ln\frac{i-k}{i-k-1} + (x_{i-1} - x_i)\left(-1 + (i-k)\ln\frac{i-k}{i-k-1}\right) \qquad (19)$$

We observed that $I_i^{(f)}$ and (17), $I_k^{(c)}$ and (18) are exactly the same, but $I_i^{(r)}$ is different from (19). This equation does influence the result of the Hilbert transform of a signal.

3.3. Method III: Convolution method.

In this method we consider complex sequences for which the real and imaginary components of analytic signal can be related through a convolution. These Hilbert transform relations are particularly useful in representing bandpass signals as complex signals.

The estimate Hilbert transformer of an analytic signal was obtained by performing a filtering operation on the analytic signal itself. The integral of Cauchy given by equation (3) can be rewritten in the form of a convolution as

$$x_i(n) = \sum_{l=-L}^{L} x_r(n-l)h(l)$$

(20)

where (20) is the desired Hilbert transform of the imaginary part of a discrete-time analytic signal. L indicate the order of filter and its maximum order is given by

$$L = Nimf - 1$$

(21)

where $Nimf$ indicates the total number of IMFs, including the residue $r(t)$. The filtering process eliminates the residue to avoid the mistake in the initial and end data points of recording.

The filter with the desired features which has the impulse response $h(n)$ of a 90-degree phase shifter is given by Oppenheimer and Schafer [6] as

$$h(n) = \frac{1}{2\pi}\int_{-\pi}^{0} je^{j\omega n} d\omega - \frac{1}{2\pi}\int_{0}^{\pi} je^{j\omega n} d\omega$$

(22)

or

$$h(n) = \begin{cases} \dfrac{2}{\pi}\dfrac{\sin^2(\pi n/2)}{n} & n \neq 0, \\ 0, & n = 0. \end{cases}$$

(23)

The real signal is decomposed in mono-component signals and on the IMF than contain the highest frequency oscillation is applied the convolution method for to have an estimation of its Hilbert transform. In this study $L = 1$ in (20) provided an adequate amplitude response and perfect 90-degree phase response.

4. EXPERIMENTAL RESULTS AND DISCUSSION

The three methods are assessed into an oscillatory electric power signal. Figure 1 shows the temporal behavior of an

active power signal, in which is considered a time step of 0.05s and the data set consist of 1217 data points corresponding to 60.8s of record data with a nonlinear and non-stationary dynamic evolution.

In the course of the evaluation, the empirical mode decomposition was applied to the power signal to obtain the IMFs using the shifting method described in the previous section and each IMF has a particular physical interpretation.

These IMFs are then employed to extract the time-varying amplitude and frequency of the intrinsic oscillations modes in the critical system.

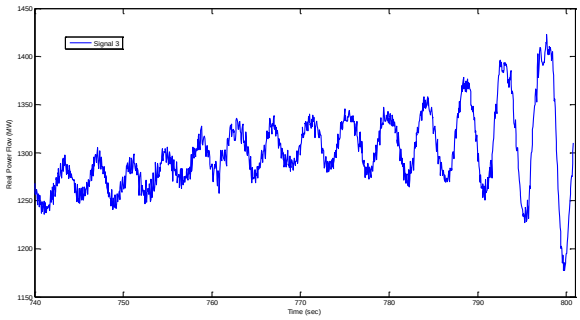


Figure 1. Active power flow of the signal under study.

Figure 2 shows the results of the application of the EMD method for to obtain the IMFs and residue of the signal under study. Is observed that the amplitude of each IMF varies, and more particularly the amplitude of IMF_4 is much greater than others IMFs.

This is checked and observed in Table 1, where the energy level of IMF4 represents 95.3% of the total energy of the original signal. Therefore the IMF_4 is the dominant IMF with the highest oscillation frequency. It is noted that IMF_1, IMF_2 and IMF_3 are pseudo-components named noise and can be filtered to reconstruct the signal.

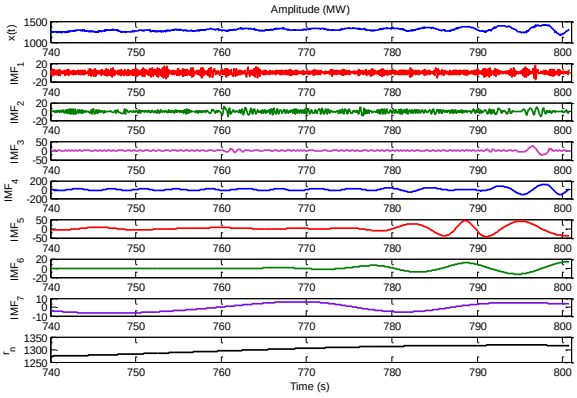


Figure 2. The whole set of IMFs and residue of the power signal

TABLE 1. Energy of each IMF

IMF1	IMF2	IMF3	IMF4	IMF5	IMF6	IMF7	Residue
0.004	0.002	0.001	0.953	0.018	0.009	0.007	0.006

The instantaneous amplitude of the first five IMFs estimated with the three proposed approaches are observed in figure 3, in which verifies that the dominant IMF is the IMF_4, which is an unstable oscillation.

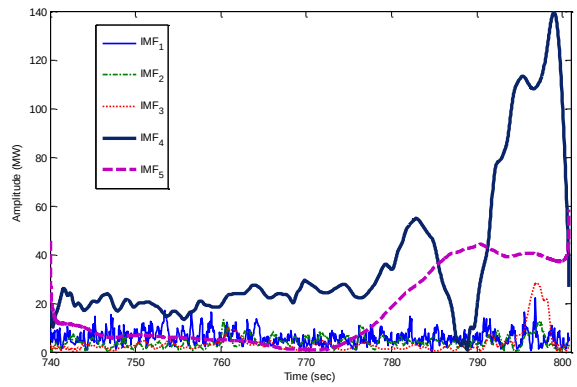


Figure 3. Instantaneous amplitude A(t) estimate of the first five IMFs.

The instantaneous amplitude of IMF_4 is compared with the three methods in the whole data point in the upper panel of figure 4, in the middle of same figure is observed the initial data point only, in which the best estimation of this instantaneous attribute is made with the Convolution method. In Addition, in the bottom of figure 4 shows the end data point and is demonstrated that the best estimation is the Convolution method because it has fewer fluctuations and peak values. The Time-domain approach and the Convolution method are very similar in the whole data-time and the Frequency-domain becomes similar to the others only between the 752.5 and 798 seconds in the time range.

The instantaneous frequency of IMF_4 is estimated by the three methods and is shown in figure 5. The amplitude of instantaneous frequency of IMF_4 increases in time and suggest the presence of strong frequency modulation.

The Time-domain approach produce a great oscillation in the initial data points (see figure 5b), in the end data points (see figure 5c) and produce important differences with the others methods proposed. These mentioned oscillations are different from the fluctuations in the time recording of instantaneous frequency, which indicate nonlinear effects into the nature of the oscillations.

The Convolution method provides a better estimate because it has not great peaks at the initial and end points of whole data. The Frequency-domain becomes similar to the other methods between the 740.15 and 800.65 seconds in the time range.

To implement the convolution filter, the residue of IMFs is

removed and the error in the initial and end data points in the recording is eliminate.

The representation of instantaneous amplitude and frequency calculated in the power system the Hilbert transformer via Convolution method have most accuracy that a filter of second or major order approximation.

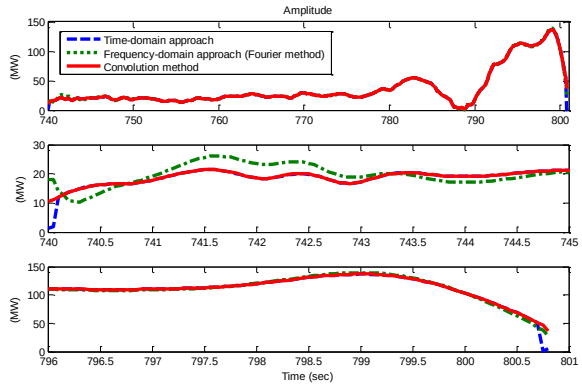
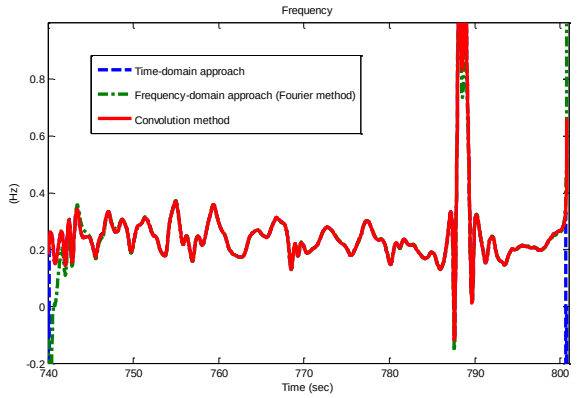
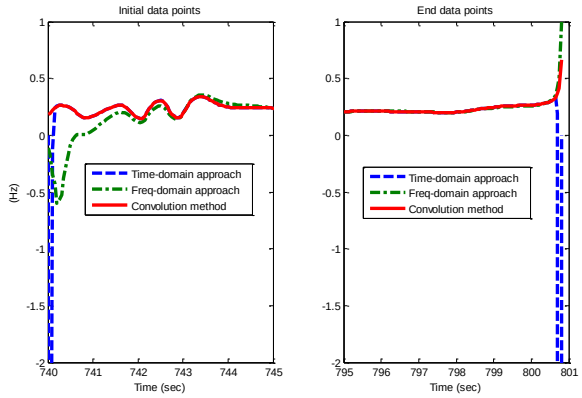


Figure 4.instantaneous amplitude A(t) of IMF$_4$
Top: whole data. Middle: initial data. Bottom: end data.



(a)



(b) (c)

Figure 5. The instantaneous frequency of IMF$_4$ derived from three methods proposed: (a) whole data, (b) initial data, (c) end data.

Compared with the Time-domain approach and the Frequency-domain, the Convolution method is an efficient approach to compute the Hilbert transform of a real power, where the maximum error between the approximation and the desired instantaneous frequency is minimized.

5. CONCLUSIONS

Three different methods for computation of the Hilbert transform have been presented and discussed. The approaches to the numerical calculation of the Hilbert transform are based on the computation of the analytic signal in nonlinear and non-stationary power system signals and the comparison is made on the instantaneous frequency of an oscillatory signal.

As demonstrated in this study, Convolution method is the most feasible way to obtain reliable dynamic behaviour of the system, whose nonlinear and non-stationary oscillations over all the data-time, require precise and accurate analysis to determine the instantaneous characteristics, mainly in the estimation of the instantaneous frequency. In this approach, filtering operation on the analytic signal eliminates noise and fluctuations in the initial and end data points and allows us to obtain a more accurate estimate of the instantaneous frequency during the analysis of the dynamics of the system under study.

Given the goods results obtained in this study, the method is expected to have various applications in dynamic analysis, including the determination of instability and the online assessment of damping. In addition, analysis can yield instantaneous parameters accurate enough to use in monitoring schemes online.

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